Midterm Exam 2 - Mechanics of Materials

Seoul National University

May 11th, 2026

Problem 1. (Modeling) Consider three types of deformation: axial loading, torsion, and bending, which are governed respectively by

$$\frac{d}{dx}\left[EA\frac{du}{dx}\right] + f = 0, \tag{1}$$

$$\frac{d}{dx}\left[GJ\frac{d\varphi}{dx}\right] + m = 0, \quad \text{and} \tag{2}$$

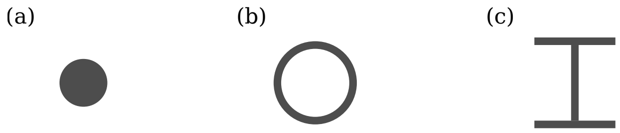
$$\frac{d^2}{dx^2}\left[EI\frac{d^2w}{dx^2}\right] - q = 0. \tag{3}$$

Here, the cross-sectional area A, the second polar moment of area J, and the second moment of area I are defined by

$$A = \int_{\mathcal{A}} dA, \quad J = \int_{\mathcal{A}} \rho^2\, dA, \quad \text{and} \quad I = \int_{\mathcal{A}} y^2\, dA. \tag{4}$$

(a) Explain the role of A, J, and I in the context of one-dimensional model reduction from a general three-dimensional elastic body.

(b) From the three cross-sections shown below, explain which cross-section is preferred for each deformation type. Assume that the cross-sectional areas are identical.



Problem 1.

Problem 2. (Strain tensor) Consider a displacement field

$$\boldsymbol{u} = (\alpha - \theta\, y)\,\hat{\boldsymbol{e}}_x + \theta\, x\,\hat{\boldsymbol{e}}_y, \tag{5}$$

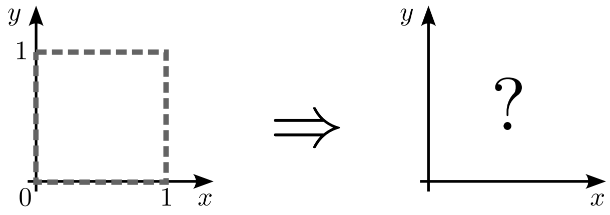
where α and θ are given real numbers and $\hat{\boldsymbol{e}}_x$ and $\hat{\boldsymbol{e}}_y$ are Cartesian basis vectors.

(a) Let $\alpha = 0.1$ and $\theta = 0.1$. Sketch the deformed shape of a unit square $(0,1)\times(0,1)$ in the xy-plane.

(b) Compute the linear strain

$$\boldsymbol{\varepsilon} := \frac{1}{2}\Big(\operatorname{grad}\boldsymbol{u} + (\operatorname{grad}\boldsymbol{u})^T\Big) \tag{6}$$

for general α and θ, and discuss the result in light of your sketch from part (a).

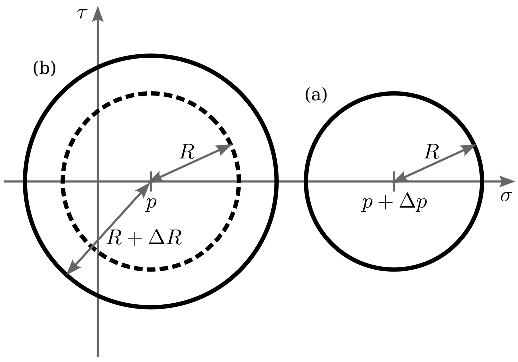


Problem 2.

Problem 3. (Mohr circle) Let $\boldsymbol{\sigma}$ be a plane stress state whose Mohr circle is centered at p with radius R. An additional plane stress $\Delta\boldsymbol{\sigma}$ is superposed so that $\boldsymbol{\sigma}_{\text{new}} = \boldsymbol{\sigma} + \Delta\boldsymbol{\sigma}$ represents the new stress state.

(a) If the new Mohr circle is shifted in its center ($p \to p + \Delta p$) without changing its radius compared to the original one, what is the characteristic of $\Delta\boldsymbol{\sigma}$?

(b) If the new Mohr circle is changed in its radius ($R \to R + \Delta R$) without shifting its center compared to the original one, what is the characteristic of $\Delta\boldsymbol{\sigma}$?



Problem 3.

Useful formula

- Stress tranformation in two-dimension (plane stress):

$$\sigma_{x'x'} = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos 2\theta + \sigma_{xy} \sin 2\theta, \tag{7}$$

$$\sigma_{y'y'} = \frac{\sigma_{xx} + \sigma_{yy}}{2} - \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos 2\theta - \sigma_{xy} \sin 2\theta, \quad \text{and} \tag{8}$$

$$\sigma_{x'y'} = -\frac{\sigma_{xx} - \sigma_{yy}}{2} \sin 2\theta + \sigma_{xy} \cos 2\theta. \tag{9}$$

- Mohr circle when $\sigma_1, \sigma_2 > 0$: