Final Exam — Mechanics of Materials

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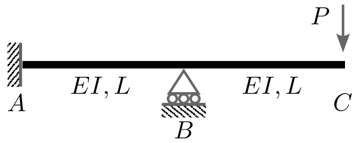
Problem 1. Consider the statically indeterminate beam shown below.



(a) State the boundary conditions and the reaction forces.

(b) Show your choice of primary structure, indicating the redundant you remove, and state the corresponding compatibility condition. Do not solve the problem.

(c) Sketch the deformed shape of the original problem, indicating which regions are linear (straight), quadratic, cubic, etc. Justify your answer using the boundary conditions and admissibility.

Problem 2. A prismatic bar of length L occupies $0 \le x \le L$ with its centroidal axis along $\boldsymbol{e}_x$. The cross-section $\mathcal{A}$ is a solid circle centered on the axis, lying in the y–z plane. The geometrical parameters are denoted by $A = \int_{\mathcal{A}} dA$, $I = \int_{\mathcal{A}} y^2\, dA$, and $J = \int_{\mathcal{A}} \rho^2\, dA$, where $\rho^2 = y^2 + z^2$ is the squared distance from the centroidal axis. The material is homogeneous, isotropic, and linearly elastic with Lamé parameters λ and μ.

The bar undergoes the displacement field

$$u_x = 0, \qquad u_y = -\alpha x z, \qquad u_z = \alpha x y, \tag{1}$$

where α is constant.

(a) Compute the strain tensor

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left[(\operatorname{grad}\boldsymbol{u}) + (\operatorname{grad}\boldsymbol{u})^T\right].$$

(b) Compute the stress tensor

$$\boldsymbol{\sigma} = 2\mu\boldsymbol{\varepsilon} + \lambda\,\operatorname{tr}(\boldsymbol{\varepsilon})\,\boldsymbol{I}.$$

(c) Calculate the strain energy

$$U = \frac{1}{2}\int_\Omega \boldsymbol{\sigma} : \boldsymbol{\varepsilon}\, d\Omega$$

in terms of A, I, J, L, λ, μ, and α (not all of these need appear). Identify the physical meaning of α.

Problem 3. Consider a simply supported beam–column of length L with flexural rigidity $EI(x)$, carrying a distributed transverse load $q(x)$ and an axial force $P(x)$ taken positive in compression. The boundary value problem reads

$$\begin{cases} \dfrac{d^2}{dx^2}\left[EI\dfrac{d^2w}{dx^2}\right] + \dfrac{d}{dx}\left[P\dfrac{dw}{dx}\right] = q, & x \in (0, L), \\ w = 0, & x = 0 \text{ and } x = L, \\ EI\dfrac{d^2w}{dx^2} = 0, & x = 0 \text{ and } x = L. \end{cases} \tag{2}$$

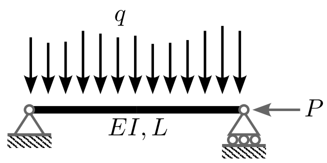
The corresponding total potential energy reads

$$\Pi\left[w\right] = \frac{1}{2}\int_0^L \left(\frac{d^2w}{dx^2}EI\frac{d^2w}{dx^2} - \frac{dw}{dx}P\frac{dw}{dx}\right)dx - \int_0^L qw\,dx. \tag{3}$$

Using the principle of minimum potential energy, $D_v\Pi[w] = 0$ for every admissible direction v, recover problem (2). Assume $v = 0$ at $x = 0$ and $x = L$. Hint: The directional derivative of a functional $\Pi[w]$ in the direction v is

$$D_v\Pi[w] = \lim_{\epsilon\to 0}\frac{\Pi[w+\epsilon v] - \Pi[w]}{\epsilon}. \tag{4}$$

Then the gradient g is obtained by writing $D_v\Pi[w] = \int_0^L g\,v\,dx$.

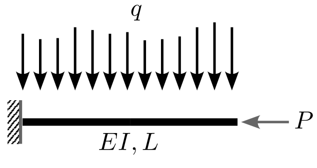


Problem 3.

Problem 4. Consider a cantilever column, fixed at $x = 0$ and free at $x = L$:

$$\begin{cases} \dfrac{d^2}{dx^2}\left[EI\dfrac{d^2w}{dx^2}\right] + \dfrac{d}{dx}\left[P\dfrac{dw}{dx}\right] = q, & x \in (0, L), \\ w = 0, & x = 0, \\ \dfrac{dw}{dx} = 0, & x = 0, \\ ?? = 0, & x = L, \\ ?? = 0, & x = L. \end{cases} \tag{5}$$

Using the same total potential energy (3), repeat the procedure to obtain the governing equation. Assume the admissible direction v satisfies $v = dv/dx = 0$ at $x = 0$. Complete problem (5); that is, determine the two natural boundary conditions at $x = L$ produced by the variational procedure.



Problem 4.