Midterm Exam 2 - Mechanics of Materials

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Problem 1. (Statically indeterminate structure) Consider a statically indeterminate structure composed of a beam and a cable (Figure 1).

1. Demonstrate that the structure is statically indeterminate by listing all reaction forces and the available equilibrium equations. Do not evaluate the reactions.
2. Write the compatiblity equation. Do not solve the equation.

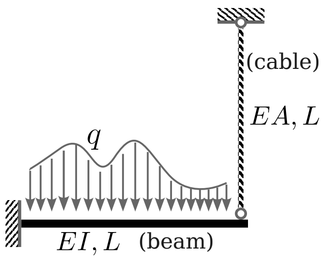

Figure 1: Beam-cable structure.

Problem 2. (Inverse source problem) A beam of span L and constant flexural rigidity EI is simply supported at its ends and carries an unknown transverse load $q(x)$ (Figure 2).

1. State the boundary conditions for the given problem.
2. Consider two trial solutions $w_1(x)$ and $w_2(x)$, which are given by

$$w_1(x) = w_o\left(x^4 - 2Lx^3 + L^2x^2\right) \quad \text{and} \tag{1a}$$
$$w_2(x) = w_o\left(x^4 - 2Lx^3 + L^3x\right). \tag{1b}$$

   Examine the admissibilities of the two trial solutions. Namely, are the above solutions physically allowed both in the domain and on the boundary?

3. Choose one solution from (1) and determine the corresponding $q(x)$.

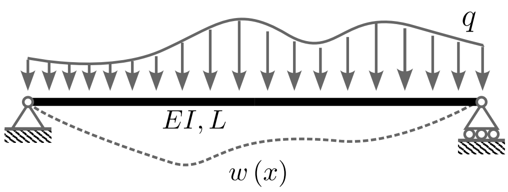

Figure 2: Simple beam.

Problem 3. (Morh's circle and stress transformation) Figure 3 depicts a plane-stress state ($\sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0$) in which the normal stresses on the reference axes are equal and the in-plane shear stress vanishes (Figure 3(a)).

1. Draw the Morh's circle for this state and determine the stress components in the rotated frame shown in Figure 3(b).
2. Consider a three-dimensional generalization of the problem, where the corresponding stress tensor reads

$$\boldsymbol{\sigma} = -p\boldsymbol{I} = \begin{bmatrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{bmatrix}. \tag{2}$$

   Investigate how this tensor behaves under an arbitrary rotation, and give a physical situation in which such a stress state occurs.

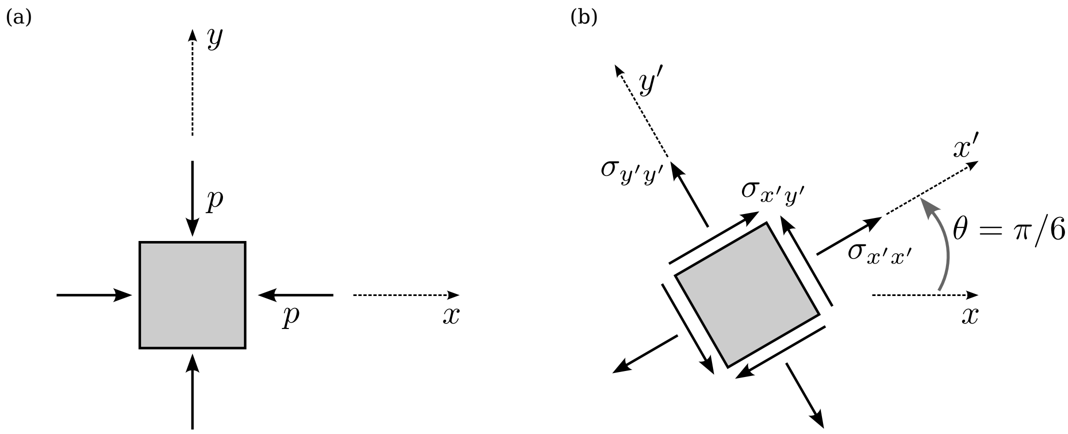

Figure 3: Stress states at (a) the reference frame. (b) at a rotated frame with $\theta = \pi/6$.

Useful formula

- Strain energy of a beam:

$$U = \frac{1}{2}\int_0^L \frac{d^2w}{dx^2} EI \frac{d^2w}{dx^2} dx = \frac{1}{2}\int_0^L \frac{M^2}{EI} dx. \tag{3}$$

- Stress transformation in general:

$$\boldsymbol{\sigma}' = \boldsymbol{R}\boldsymbol{\sigma}\boldsymbol{R}^T, \tag{4}$$

where $R_{ij} = \boldsymbol{e}_{x'_i} \cdot \boldsymbol{e}_{x_j}$ is the proper rotation matrix such that $\boldsymbol{R}^T = \boldsymbol{R}^{-1}$ and $\det \boldsymbol{R} = 1$. $\boldsymbol{e}_{x'_i}$ and $\boldsymbol{e}_{x_j}$ are orthnormal basis vectors.

- Stress tranformation in two-dimension (plain stress):

$$\sigma_{x'x'} = \frac{\sigma_{xx}+\sigma_{yy}}{2} + \frac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\theta + \sigma_{xy}\sin 2\theta, \tag{5}$$

$$\sigma_{y'y'} = \frac{\sigma_{xx}+\sigma_{yy}}{2} - \frac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\theta - \sigma_{xy}\sin 2\theta, \quad \text{and} \tag{6}$$

$$\sigma_{x'y'} = -\frac{\sigma_{xx}-\sigma_{yy}}{2}\sin 2\theta + \sigma_{xy}\cos 2\theta. \tag{7}$$

- Morh circle when $\sigma_1, \sigma_2 > 0$: