Midterm Exam 2 - Structural Analysis 1

Seoul National University

November 17th, 2025

Problem 1. Consider a beam with a small initial deflection w_o due to a fabrication imperfection. The total deflection w_{tot} is expressed as the sum of the initial deflection w_o and the additional deflection w induced by external loads and boundary conditions:

$$w_{\text{tot}} = w + w_o. \tag{1}$$

Assume that the initial deflection does not induce strain energy and therefore generates no internal forces. Accordingly, the bending moment and shear force depend only on w:

$$M = EI\frac{d^2w}{dx^2} \quad \text{and} \quad V = \frac{d}{dx}\left[EI\frac{d^2w}{dx^2}\right]. \tag{2}$$

The external load q must likewise be balanced solely by w, so that

$$\frac{d^2}{dx^2}\left[EI\frac{d^2w}{dx^2}\right] = q. \tag{3}$$

Then, a beam with an initial deflection w_o and no external load ($q = 0$) is governed by

$$\begin{cases} \dfrac{d^2}{dx^2}\left[EI\dfrac{d^2w}{dx^2}\right] = \dfrac{d^2}{dx^2}\left[EI\dfrac{d^2\left(w_{\text{tot}} - w_o\right)}{dx^2}\right] = 0, & x \in (0, L) \\ w_{\text{tot}} = 0, & x = 0 \\ w_{\text{tot}} = 0, & x = L \\ \dfrac{dw_{\text{tot}}}{dx} = 0, & x = 0 \\ \dfrac{dw_{\text{tot}}}{dx} = 0, & x = L \end{cases}. \tag{4}$$

(a) Show that $w_{\text{tot}} = 0$ for any quadratic initial deflection

$$w_o = ax^2 + bx + c, \tag{5}$$

where a, b, c are arbitrary constants.

(b) Sketch the shear force and bending moment diagrams corresponding to the quadratic initial deflection.

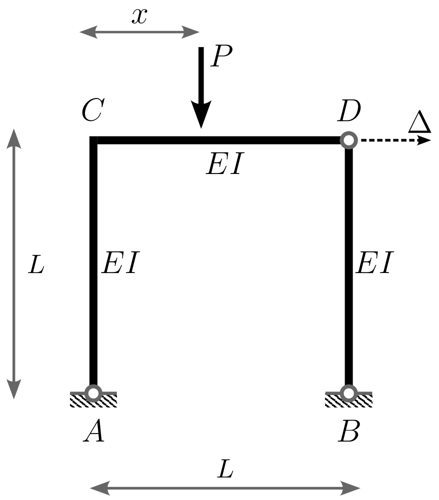
(c) Explain the analogy between this problem and a beam subjected to a linear temperature gradient along the cross-sectional depth.

Problem 2. Consider a three-hinged portal frame subjected to a downward point load P applied on the horizontal member CD at an arbitrary position x, where $0 < x < L$.

(a) Sketch the bending moment diagram.

(b) Determine the resulting horizontal displacement Δ at node D, taking the positive direction to be to the right.

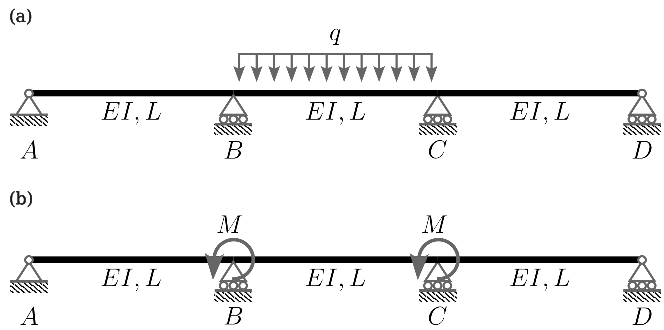
(c) Identify whether there exists a position x for which $\Delta < 0$, and justify your conclusion.



Problem 2.

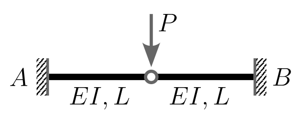
Problem 3. Consider a three-span continuous beam subjected to two different loading cases. Let $m_{(a)}$ and $m_{(b)}$ denote the bending moments associated with cases (a) and (b), respectively. Evaluate

$$\int_0^{3L} \frac{m_{(a)} m_{(b)}}{EI} dx. \tag{6}$$



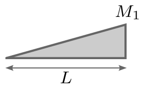
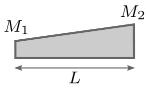
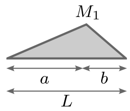
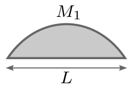
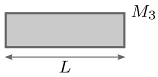
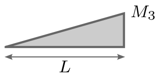
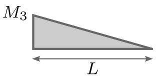
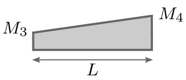
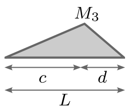
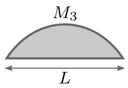
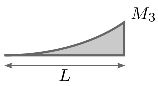
Problem 3.

Problem 4. Consider a beam with an internal hinge located at midspan. Determine the bending moment reaction at support A.



Problem 4.

Integration table.

M_A / M_B				
	$\frac{L}{2} M_1 M_3$	$\frac{L}{2}(M_1+M_2) M_3$	$\frac{L}{2} M_1 M_3$	$\frac{2L}{3} M_1 M_3$
	$\frac{L}{3} M_1 M_3$	$\frac{L}{6}(M_1+2M_2) M_3$	$\frac{L}{6}\left(1+\frac{a}{L}\right) M_1 M_3$	$\frac{L}{3} M_1 M_3$
	$\frac{L}{6} M_1 M_3$	$\frac{L}{6}(2M_1+M_2) M_3$	$\frac{L}{6}\left(1+\frac{b}{L}\right) M_1 M_3$	$\frac{L}{3} M_1 M_3$
	$\frac{L}{6} M_1 (M_3+2M_4)$	$\frac{L}{6} M_1 (2M_3+M_4)$ $+\frac{L}{6} M_2 (M_3+2M_4)$	$\frac{L}{6}\left(1+\frac{b}{L}\right) M_1 M_3$ $+\frac{L}{6}\left(1+\frac{a}{L}\right) M_1 M_4$	$\frac{L}{3} M_1 (M_3+M_4)$
	$\frac{L}{6}\left(1+\frac{c}{L}\right) M_1 M_3$	$\frac{L}{6}\left(1+\frac{d}{L}\right) M_1 M_3$ $+\frac{L}{6}\left(1+\frac{c}{L}\right) M_2 M_3$	for $c \le a$, $\frac{L}{3} M_1 M_3$ $-\frac{L(a-c)^2}{6ad} M_1 M_3$	$\frac{L}{3}\left(1+\frac{cd}{L^2}\right) M_1 M_3$
	$\frac{L}{3} M_1 M_3$	$\frac{L}{3}(M_1+M_2) M_3$	$\frac{L}{3}\left(1+\frac{ab}{L^2}\right) M_1 M_3$	$\frac{8L}{15} M_1 M_3$
	$\frac{L}{4} M_1 M_3$	$\frac{L}{12}(M_1+3M_2) M_3$	$\frac{L}{12}\left(1+\frac{a}{L}+\frac{a^2}{L^2}\right) M_1 M_3$	$\frac{L}{5} M_1 M_3$