Midterm Exam 2 - Structural Analysis 1

Seoul National University

November 18th, 2024

Problem 1. Calculate the downward displacement at B for each system shown in Figure 1(a), (b), and (c). All beam members have a flexural rigidities of EI and a length of $2L$, while all truss members have an axial stiffness of EA.

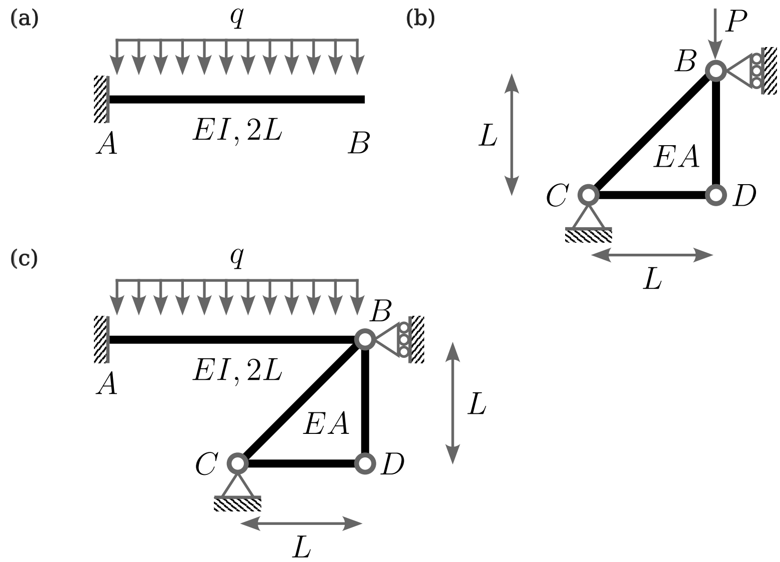


Figure 1: Problem 1.

Problem 2. Calculate the bending moment at B for each system shown in Figure 2(a) and (b). All beam members have a flexural rigidity of EI and a length of L. In system (b), there is a downward support settlement of Δ at point B.

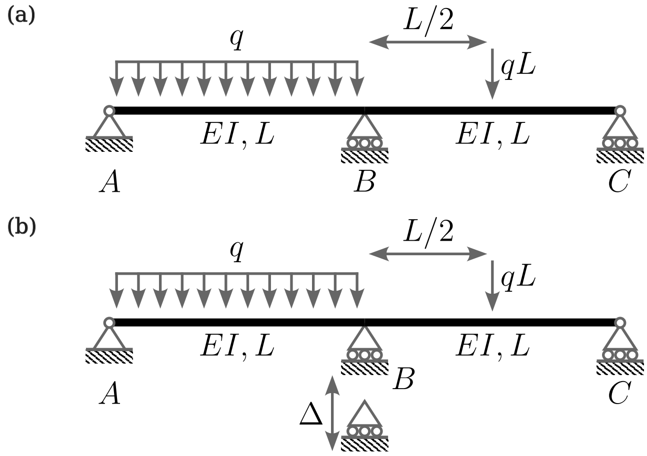


Figure 2: Problem 2.

Problem 3. Calculate the order of indeterminancy for the structure shown in Figure 3 counting the number of unknowns and equations.

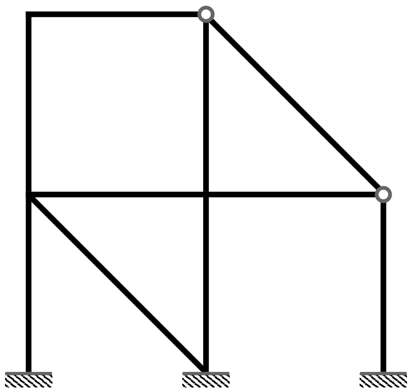


Figure 3: Problem 3.

Problem 4. The governing equation for one-dimensional axial deformation is given by

$$\frac{d}{dx}\left[EA\frac{du}{dx}\right] + f = 0, \quad x \in (0, L), \tag{1}$$

where u denotes the axial deformation, $F = EA(du/dx)$ denotes the axial force, and f is the external force within the domain.

(a) Using the weighted residual method, derive a general expression for the principle of virtual work $\delta W_{\text{int}} = \delta W_{\text{ext}}$, as follows:

$$\underbrace{\int_0^L \frac{\overline{F}F}{EA} dx}_{=\delta W_{\text{int}}} = \underbrace{[\overline{u}F]_{x=0}^{x=L} + \int_0^L \overline{u} f \, dx}_{=\delta W_{\text{ext}}}. \tag{2}$$

(b) Then, the reciprocity implies

$$\int_0^L \frac{\overline{F}F}{EA} dx = \left[u\overline{F}\right]_{x=0}^{x=L} + \int_0^L u\overline{f} dx. \tag{3}$$

Starting from (3) and applying the assumptions for a truss, derive the expression for the principle of virtual work for the truss shown in Figure 4, namely,

$$\Delta\overline{F} = \frac{\overline{F}FL}{EA}, \tag{4}$$

where Δ is the displacement at B.

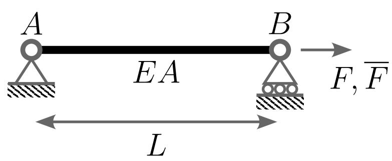
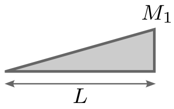
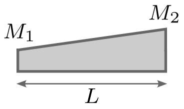
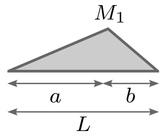
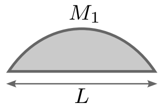
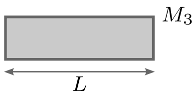
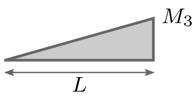
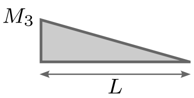
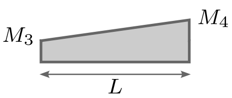
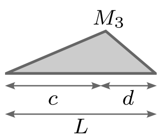
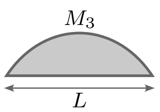
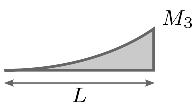


Figure 4: Problem 4(b).

Table 1: Integration formulae for a product of two functions $\int_0^L M_A M_B dx$.

M_B \ M_A				
	$\frac{L}{2} M_1 M_3$	$\frac{L}{2}(M_1+M_2)M_3$	$\frac{L}{2} M_1 M_3$	$\frac{2L}{3} M_1 M_3$
	$\frac{L}{3} M_1 M_3$	$\frac{L}{6}(M_1+2M_2)M_3$	$\frac{L}{6}\left(1+\frac{a}{L}\right) M_1 M_3$	$\frac{L}{3} M_1 M_3$
	$\frac{L}{6} M_1 M_3$	$\frac{L}{6}(2M_1+M_2)M_3$	$\frac{L}{6}\left(1+\frac{b}{L}\right) M_1 M_3$	$\frac{L}{3} M_1 M_3$
	$\frac{L}{6} M_1 (M_3+2M_4)$	$\frac{L}{6} M_1 (2M_3+M_4)$ $+\frac{L}{6} M_2 (M_3+2M_4)$	$\frac{L}{6}\left(1+\frac{b}{L}\right) M_1 M_3$ $+\frac{L}{6}\left(1+\frac{a}{L}\right) M_1 M_4$	$\frac{L}{3} M_1 (M_3+M_4)$
	$\frac{L}{6}\left(1+\frac{c}{L}\right) M_1 M_3$	$\frac{L}{6}\left(1+\frac{d}{L}\right) M_1 M_3$ $+\frac{L}{6}\left(1+\frac{c}{L}\right) M_2 M_3$	for $c \leq a$, $\frac{L}{3} M_1 M_3$ $-\frac{L(a-c)^2}{6ad} M_1 M_3$	$\frac{L}{3}\left(1+\frac{cd}{L^2}\right) M_1 M_3$
	$\frac{L}{3} M_1 M_3$	$\frac{L}{3}(M_1+M_2)M_3$	$\frac{L}{3}\left(1+\frac{ab}{L^2}\right) M_1 M_3$	$\frac{8L}{15} M_1 M_3$
	$\frac{L}{4} M_1 M_3$	$\frac{L}{12}(M_1+3M_2)M_3$	$\frac{L}{12}\left(1+\frac{a}{L}+\frac{a^2}{L^2}\right) M_1 M_3$	$\frac{L}{5} M_1 M_3$