Final Exam - Structural Analysis 1

Seoul National University

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Problem 1. Consider the quarter-circular structure shown in Figure 1 ($x^2 + y^2 = R^2$, $x, y \in (0, R)$), subjected to two different loading cases. Derive the relation between P, Δ, M, and θ using the general expression of the reciprocity:

$$\int_0^l q_1 w_2 ds - [V_1 w_2]_0^l + [M_1 \theta_2]_0^l = \int_0^l q_2 w_1 ds - [V_2 w_1]_0^l + [M_2 \theta_1]_0^l . \quad (1)$$

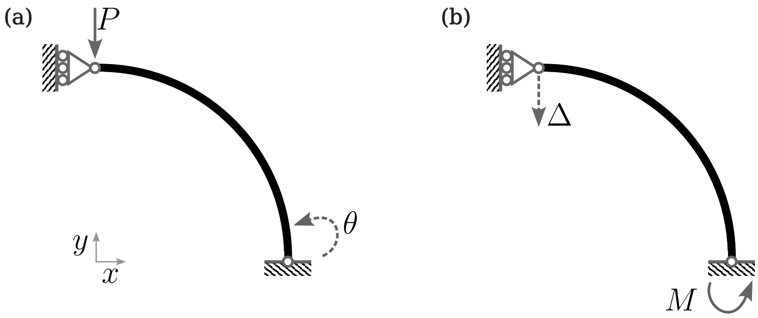


Figure 1: Reciprocity. (a) Loading case 1. (b) Loading case 2.

Problem 2. Consider a cantilever beam with a spring support (Figure 2). The corresponding governing equation and the boundary conditions are

$$\begin{cases} \dfrac{d^2}{dx^2}\left[EI\dfrac{d^2w}{dx^2}\right] = q, & x \in (0, L) \\ w = 0, & x = 0 \\ \dfrac{dw}{dx} = 0, & x = 0 \\ EI\dfrac{d^2w}{dx^2} = 0, & x = L \\ EI\dfrac{d^3w}{dx^3} - kw = 0, & x = L \end{cases} . \quad (2)$$

(a) Let $\overline{w}$ denote a test function (or the virtual displacement), derive the Principle of Virtual Work using the weighted residual method, as follows:

$$\int_0^L \frac{d^2\overline{w}}{dx^2} EI \frac{d^2w}{dx^2} dx + wk\overline{w}|_{x=L} - \int_0^L \overline{w} q dx = 0. \quad (3)$$

(b) State the corresponding essential (or Dirichlet) boundary conditions for $\overline{w}$ at $x = 0$.

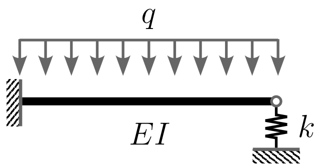


Figure 2: Robin boundary condition.

Problem 3. Consider an infinitely repeated series of identical beam members subjected to a uniform distributed load q (Figure 3). By exploiting symmetry, calculate the bending moment at any support.

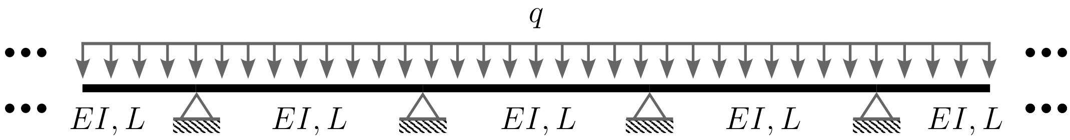


Figure 3: Indeterminate structure.

Problem 4. Consider an infinitely repeated series of identical beam members subjected to a point load at $x = \xi$, $x, \xi \in (-\infty, \infty)$ (Figure 4).

(a) Informed by the Müller-Breslau's principle (without conducting any numerical calculations), sketch the influence line for the support reaction at $x = 0$ when the point load is placed at an arbitrary position ξ.

(b) Let $I_R(\xi)$ denote the influence line in part (a). What is the value of $I_R(\xi)$ at $\xi = 0$?

(c) Determine the limiting value of the influence line as ξ tends to infinity, i.e., find $\lim_{\xi \to \infty} I_R(\xi)$.

(d) Evaluate the integral of the influence line over the entire real line, i.e., find $\int_{-\infty}^{\infty} I_R(\xi)\, d\xi$.

Note: Please provide appropriate reasoning for each problem.

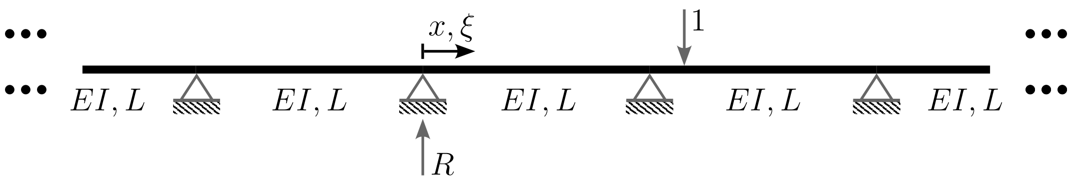
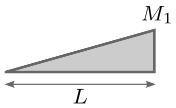
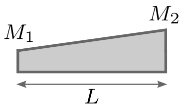
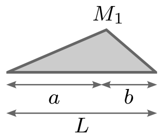
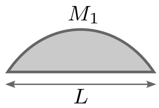
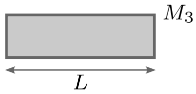
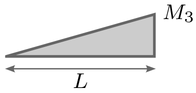
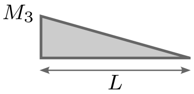
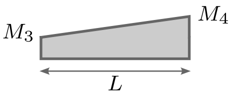
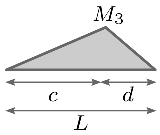
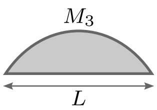
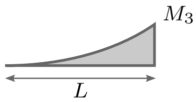


Figure 4: Influence line.

Acknowledgment Thank you all for participating in this class. Life as an engineer revolves around identifying and solving problems, often without clear or predefined solutions. Throughout this course, I tried to share my perspective on self-verifying solutions rather than focusing solely on computational skills. I hope you found this exploration of structural analysis engaging and that it inspires you to continue your studies in Structural Analysis 2 and beyond.

Table 1: Integration formulae for a product of two functions $\int_0^L M_A M_B dx$.

M_A \ M_B				
	$\frac{L}{2} M_1 M_3$	$\frac{L}{2}\left(M_1 + M_2\right) M_3$	$\frac{L}{2} M_1 M_3$	$\frac{2L}{3} M_1 M_3$
	$\frac{L}{3} M_1 M_3$	$\frac{L}{6}\left(M_1 + 2M_2\right) M_3$	$\frac{L}{6}\left(1 + \frac{a}{L}\right) M_1 M_3$	$\frac{L}{3} M_1 M_3$
	$\frac{L}{6} M_1 M_3$	$\frac{L}{6}\left(2M_1 + M_2\right) M_3$	$\frac{L}{6}\left(1 + \frac{b}{L}\right) M_1 M_3$	$\frac{L}{3} M_1 M_3$
	$\frac{L}{6} M_1 \left(M_3 + 2M_4\right)$	$\frac{L}{6} M_1 \left(2M_3 + M_4\right) + \frac{L}{6} M_2 \left(M_3 + 2M_4\right)$	$\frac{L}{6}\left(1 + \frac{b}{L}\right) M_1 M_3 + \frac{L}{6}\left(1 + \frac{a}{L}\right) M_1 M_4$	$\frac{L}{3} M_1 \left(M_3 + M_4\right)$
	$\frac{L}{6}\left(1 + \frac{c}{L}\right) M_1 M_3$	$\frac{L}{6}\left(1 + \frac{d}{L}\right) M_1 M_3 + \frac{L}{6}\left(1 + \frac{c}{L}\right) M_2 M_3$	for $c \leq a$, $\frac{L}{3} M_1 M_3 - \frac{L\left(a - c\right)^2}{6ad} M_1 M_3$	$\frac{L}{3}\left(1 + \frac{cd}{L^2}\right) M_1 M_3$
	$\frac{L}{3} M_1 M_3$	$\frac{L}{3}\left(M_1 + M_2\right) M_3$	$\frac{L}{3}\left(1 + \frac{ab}{L^2}\right) M_1 M_3$	$\frac{8L}{15} M_1 M_3$
	$\frac{L}{4} M_1 M_3$	$\frac{L}{12}\left(M_1 + 3M_2\right) M_3$	$\frac{L}{12}\left(1 + \frac{a}{L} + \frac{a^2}{L^2}\right) M_1 M_3$	$\frac{L}{5} M_1 M_3$